

# Relational Algebra Equivalences

1. All joins and products are commutative.

- $R \times S = S \times R$
- $R \bowtie_{\theta} S = S \bowtie_{\theta} R$
- $R \ltimes S = S \ltimes R$

2. Joins and products are associative

- $(R \times S) \times T = R \times (S \times T)$
- $(R \bowtie_{\theta} S) \bowtie_{\theta} T = R \bowtie_{\theta} (S \bowtie_{\theta} T)$
- $(R \ltimes S) \ltimes T = R \ltimes (S \ltimes T)$

3. Select is commutative

- $\sigma_p (\sigma_q (R)) = \sigma_q (\sigma_p (R))$

4. Conjunctive selects can cascade into individual selects

- $\sigma_{p \& q \& \dots \& z} (R) = (\sigma_p (\sigma_q (\dots (\sigma_z (R)) \dots)))$

5. Successive projects can be reduced to the final project.

- If  $list_1, list_2, \dots, list_n$  are lists of attribute names and each of the  $list_i$  contains  $list_{i-1}$ ,  
then  $\Pi_{list_1} (\Pi_{list_2} (\dots \Pi_{list_n} (R) \dots)) = \Pi_{list_1} (R)$
- So only the last project has to be executed

6. Select and project sometimes commute

- If  $p$  involves only the attributes in  $projlist$ , then select and project commute
- $\Pi_{projlist} (\sigma_p (R)) = \sigma_p (\Pi_{projlist} (R))$

7. Select and join (or product) sometimes commute

- If  $p$  involves only attributes of one of the tables being joined, then select and join commute
- $\sigma_p (R \ltimes S) = (\sigma_p (R)) \ltimes S$
- Only if  $p$  refers just to  $R$

8. Select sometimes distributes over join (or product)

- For  $p$  AND  $q$ , where  $p$  involves only the attributes of  $R$  and  $q$  only the attributes of  $S$  the select distributes over the join
- $\sigma_{p \text{ AND } q} (R \bowtie S) = (\sigma_p (R)) \bowtie (\sigma_q (S))$

9. Project sometimes distributes over join (or product)

- If projlist can be split into separate lists, list1 and list2, so that list1 contains only attributes of  $R$  and list2 contains only attributes of  $S$ , then
- $\Pi_{\text{projlist}} (R \bowtie S) = (\Pi_{\text{list1}} (R)) \bowtie (\Pi_{\text{list2}} (S))$

10. Union and intersection are commutative

- $R \cup S = S \cup R$
- $R \cap S = S \cap R$
- Set difference is not commutative.

11. Union and intersection are individually associative.

- $(R \cup S) \cup T = R \cup (S \cup T)$
- $(R \cap S) \cap T = R \cap (S \cap T)$
- Set difference is not associative

12. Select distributes over union, intersection, and difference

- $\sigma_p (R \cup S) = \sigma_p (R) \cup \sigma_p (S)$
- $\sigma_p (R \cap S) = \sigma_p (R) \cap \sigma_p (S)$
- $\sigma_p (R - S) = \sigma_p (R) - \sigma_p (S)$

13. Project distributes over union, intersection, and difference

- $\Pi_{\text{projlist}} (R \cup S) = (\Pi_{\text{projlist}} (R)) \cup (\Pi_{\text{projlist}} (S))$
- $\Pi_{\text{projlist}} (R \cap S) = (\Pi_{\text{projlist}} (R)) \cap (\Pi_{\text{projlist}} (S))$
- $\Pi_{\text{projlist}} (R - S) = (\Pi_{\text{projlist}} (R)) - (\Pi_{\text{projlist}} (S))$

14. Project is idempotent-repeating it produces the same result

- $\Pi_{\text{projlist}} (R) (\Pi_{\text{projlist}} (R)) = \Pi_{\text{projlist}} (R)$

15. Select is idempotent

- $\sigma_p (\sigma_p (R)) = \sigma_p (R)$